$\boxed{1}$

9

(1) $\sin 3\theta = \sin(\theta + 2\theta)$ ③

$= \sin\theta \cdot \cos 2\theta + \cos\theta \cdot \sin 2\theta$

$= \sin\theta(1 - 2\sin^2\theta) + \cos\theta \cdot 2\sin\theta\cos\theta$

$= \sin\theta - 2\sin^3\theta + 2\sin\theta(1 - \sin^2\theta)$

$= 3\sin\theta - 4\sin^3\theta$ ③

$\sin\theta = \frac{1}{3}$ より

$\sin 3\theta = 3 \cdot \frac{1}{3} - 4 \cdot \left(\frac{1}{3}\right)^3$

$= 1 - \frac{4}{27} = \frac{23}{27}$ ③

8

(2) $\tan^2\theta + \frac{1}{\tan^2\theta} = \left(\tan\theta + \frac{1}{\tan\theta}\right)^2 - 2 \cdot \tan\theta \cdot \frac{1}{\tan\theta}$

$= \left(\tan\theta + \frac{1}{\tan\theta}\right)^2 - 2$ ②

ここで、

$\tan\theta + \frac{1}{\tan\theta} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$ ②

よって、

$(\sin\theta + \cos\theta)^2 = \frac{1}{5}$ ②

$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = \frac{1}{5}$

$\sin\theta\cos\theta = -\frac{2}{5}$

$\therefore \tan\theta + \frac{1}{\tan\theta} = -\frac{5}{2}$

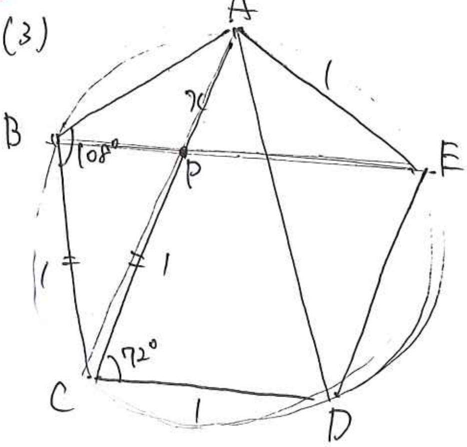
$\therefore \tan^2\theta + \frac{1}{\tan^2\theta} = \left(-\frac{5}{2}\right)^2 - 2$

$= \frac{25}{4} - 2$

$= \frac{17}{4}$ ②

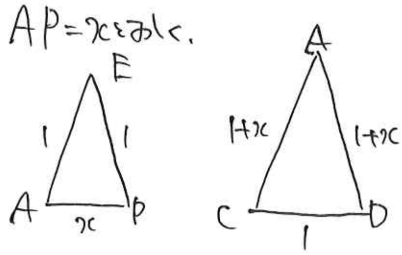
8

(3)



1辺が1の正五角形について.

AP＝xとおく.



②

上2つの三角形は相似なので、

$1 : x = 1 + x : 1.$

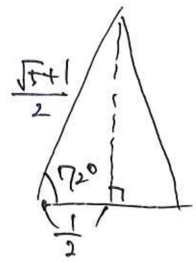
$x(x+1) = 1$

$x^2 + x - 1 = 0$

$x = \frac{-1 \pm \sqrt{5}}{2}$

$\therefore AP = \frac{\sqrt{5} - 1}{2}$

$\therefore AC = \frac{\sqrt{5} + 1}{2}$ ③



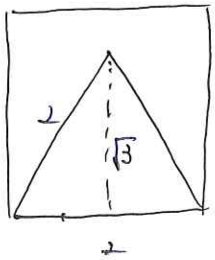
左図より.

$\cos 72^\circ = \frac{1}{\sqrt{5} + 1}$

$= \frac{\sqrt{5} - 1}{4}$ ④

2

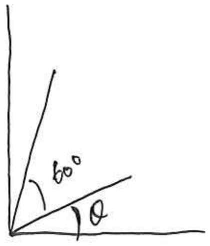
(1) 5.



求める三角形の高さは $\sqrt{3}$ となので

$$S = 2 \times \sqrt{3} \times \frac{1}{2} = \sqrt{3}$$ //

(2) 6

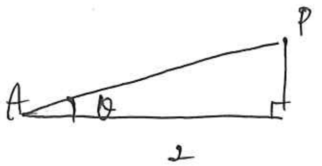


∠DAQ と ∠BAP の対称性から.

$0 \leq \theta \leq 15°$. //

($0 \leq \theta < 30°$ も可)

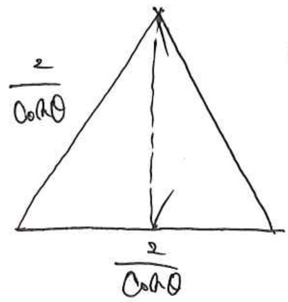
(3) 6



$$\frac{2}{AP} = \cos\theta$$

$$\therefore AP = \frac{2}{\cos\theta}$$ //

(4) 5 △APQ は1辺が $\frac{2}{\cos\theta}$ の正三角形.



$(\text{高さ}) = \frac{\sqrt{3}}{\cos\theta}$

$$\therefore S = \frac{2}{\cos\theta} \times \frac{\sqrt{3}}{\cos\theta} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{\cos^2\theta}$$ //

(5) 3 $S = \frac{\sqrt{3}}{\cos^2\theta}$ より.

Sが最大値をとるとき、$\cos^2\theta$ は最小値をとる.

$0 \leq \theta \leq 15°$ での $\cos^2\theta$ の最小値は $\theta = 15°$ のとき. ②

よって. $\cos 15° = \cos(45° - 30°)$

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\therefore S_{max} = \sqrt{3} \cdot \left(\frac{4}{\sqrt{6}+\sqrt{2}}\right)^2$$

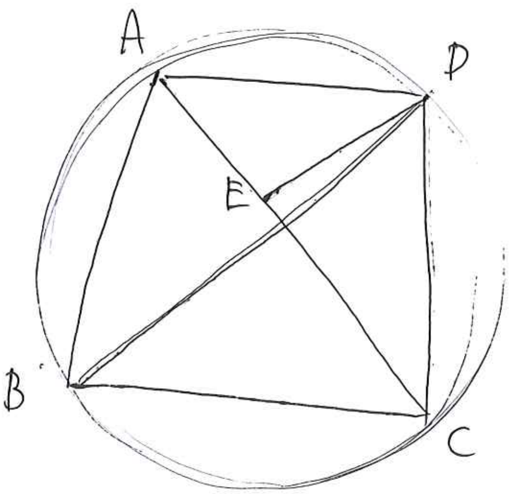
$$= \sqrt{3} \cdot 4^2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2 \cdot \left(\frac{1}{\sqrt{3}+1}\right)^2$$

$$= \sqrt{3} \cdot \cancel{8}^{4} \cdot \frac{1}{\cancel{4}_{2} + \cancel{2}\sqrt{3}}$$

$$= 4\sqrt{3} \cdot \frac{1}{2+\sqrt{3}}$$

$$= 8\sqrt{3} - 12$$ //

3



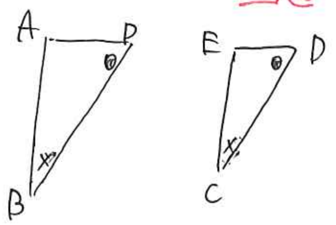
10

(1) 円周角の定理.

$\angle ABD = \angle ECD$ 3

条件より

$\angle CDE = \angle ADB$ 3



上図より $\triangle ABD \backsim \triangle ECD$

10

(2) 円周角の定理.

$\angle DAE = \angle DBC$. 3

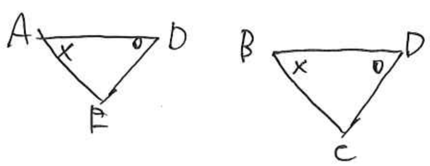
また,

$\angle ADE = \boxed{\angle ADB} - \angle EDB$.

$\angle CDB = \boxed{\angle CDE} - \angle EDB$. 3

ここで, $\angle ADB = \angle CDE$ なので

$\angle ADE = \angle CDB$. 3



上図より $\triangle ADE \backsim \triangle BDC$

5

(3).

(1)の図において,

$AB : BD = CE : CD$ が成立

$\therefore AB \cdot CD = BD \cdot CE$ ——①

(2)の図において,

$AD : AE = BD : BC$ が成立.

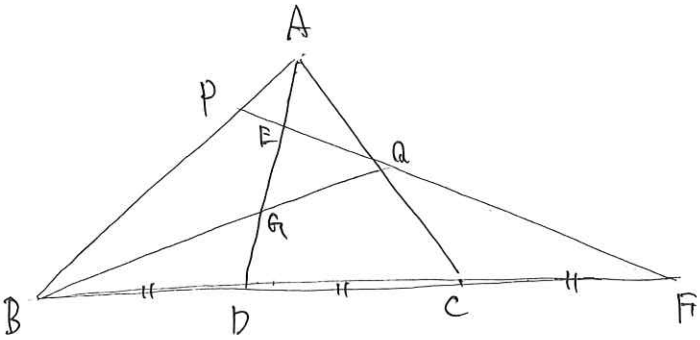
$\therefore AD \cdot BC = AE \cdot BD$ ——②

①, ② 辺をたして

$$AB \cdot CD + AD \cdot BC = BD \cdot CE + AE \cdot BD$$
$$= BD \cdot (AE + EC)$$
$$= BD \cdot AC$$

$$\therefore AB \cdot CD + AD \cdot BC = AC \cdot BD$$

4

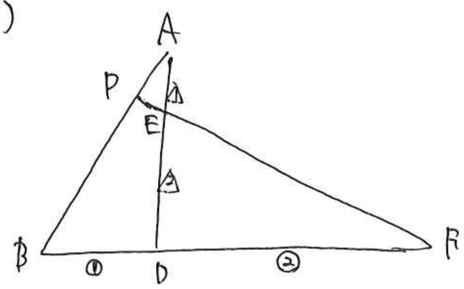


(1) $AE:ED=1:2$

Gが重心より $AG:GD=2:1$　2

$\therefore AE:EG=1:1$　5

(2)



メネラウスの定理より

$$\frac{PB}{AP}\times\frac{2}{3}\times\frac{1}{2}=1.$$

$$\frac{PB}{AP}=\frac{3}{1}\quad\therefore AP:PB=1:3$$ 3

よって $AB=4$ より $AP=1$　7

線分BQは△ABCの重心Gを通る.

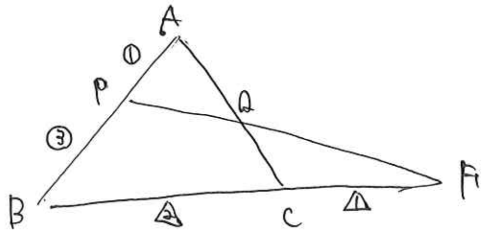
i.e. QはACの中点

$$\therefore AQ=\frac{3}{2}$$ 7

(3) (2)で用いた図で, メネラウスの定理.

$$\frac{EP}{FE}\cdot\frac{4}{1}\cdot\frac{2}{1}=1$$

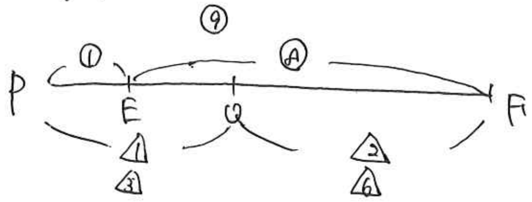
$$\frac{EP}{FE}=\frac{1}{8}\quad\therefore EP:FE=1:8.$$ 1



上図でメネラウスの定理.

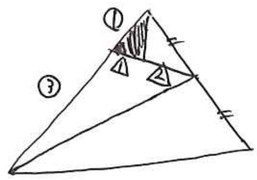
$$\frac{QP}{FQ}\cdot\frac{4}{1}\cdot\frac{1}{2}=1.$$

$$\frac{QP}{FQ}=\frac{1}{2}\quad\therefore QP:FQ=1:2$$ 1

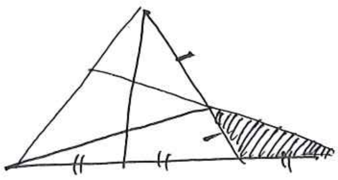


上図より $PE:EQ:QF=1:2:6$　1

(4) $\triangle ABC=\frac{1}{2}\times4\times3=6$　1



$$\triangle APE=6\times\frac{1}{2}\times\frac{1}{4}\times\frac{1}{3}=\frac{1}{4}$$ 1



$$\triangle CQF=6\times\frac{1}{2}\times\frac{1}{2}=\frac{3}{2}$$ 1